Decomposable Solution Paradigm for Uncertainty-based Transmission and Distribution Coordinated Economic Dispatch

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***Abstract*—This paper proposes a two-stage formulation and its solution paradigm for the coordinated economic dispatch of transmission and distribution network. The first stage concerns the benefits for a transmission system operator (TSO), which transfers the optimal power injection with locational marginal prices (LMPs) via the boundary node to the active distribution system operators (DSOs), whereas DSOs in the second stage return their optimal nodal power demand back to the TSO. In the distribution network, not only conventional dispatchable distributed generators but the uncertainties of renewable energy and volatile demands are considered, which are tackled via stochastic programming approach in the second stage. Two convexified *DistFlow* formulations for the branch flow in a DSO are tested and compared in this work. The proposed framework can be effectively solved by a generalized multi-cut L-shaped method. Numerical experiments on test systems support the efficacy of this paradigm.**

I. Nomenclature

1) Sets:

T / D	Transmission/distribution system.
G^T / G^D	Conventional thermal generator of transmission/distribution system.
R / I	Renewable generator/connected bus.
L^T / L^D	Load of transmission/distribution system.
F^T / F^D	Line of transmission/distribution system.
S / M	Sending/receiving bus of line.
C	Generator or injection mapping with bus.
H	Time horizon.
K	Piecewise linear segment.

2) Indices:

g / r / ℓ	Conventional generator / renewable generator / load.
f	Line.
n	Bus.
i	boundary bus/power injection.
ω	Uncertainty scenario.
k	Piecewise linear segment.
o	Iteration counter.
d	Index for multiple distribution systems.
h	Time index.

3) Parameters:

PC	Penalty cost.
r / x / b	Resistance/reactance/susceptance.
RD / RU	Ramp-down/up limits.

4) Uncertain Parameters:

P_r / P_{ℓ^D}	Renewable generation/distribution load.

5) Variables:

P_g / P_i	Active generation output/power injection.
Q_g / Q_i	Reactive generation output/power injection.
P_f / Q_f	Active/reactive line flow.
P_f^{loss} / Q_f^{loss}	Active/reactive line loss.
V / δ	Bus voltage magnitude/angle.
s_ℓ	load shedding.

II. Introduction

The coordination between transmission and distribution networks (T-D coordination) has raised great attention these days along with the wild growth of smart grids. Some recent scholars [1] suggested to adopt decentralized transactive optimization method to deal with the coordination between the TSO and DSOs, for the convenience it provides to let each operator focus only on its own management and the interfaces, which befriends the nature of hierarchical market structure.

Economic dispatch (ED) problem is thus expanded from transmission perspective to T-D coordination in a decentralized manner. Z. Li *et al.* developed a distributed algorithm to deal with this kind of problem in [2] and [3], with computational analysis on the AC optimal power flow in T-D coordination [4]. However, upon using their strategy, it is inconvenient to integrate uncertainties such as renewable energy and demand response in the distribution level, which becomes a more and more practical issue in today's power system.

To cope with the uncertainty, currently, there are two mainstream and mature techniques, namely stochastic programming (SP) and robust optimization (RO). SP uses scenario set generated from *a priori* probability distribution to represent the uncertainty, whereas RO defines conservative bounds for uncertainties to find the worst-case scenario and make decisions

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accordingly. For the uncertainty of the renewable generation and loads, extracting SP method can yield better benefit compared with using RO. Another method, distributionally robust optimization (DRO), can combine both advantages for SP and RO, which, however, asks for much more difficult implementation for decomposition framework to the coordination between TSO and DSOs *e.g.* cutting plane formulation. The exploration of using DRO technique in the coordinated ED can be one of the future works.

Apropos of the formulation of AC active/reactive power flow in the distribution system, the coordination between transmission system and secondary distribution system, which can be fairly regarded as a single-phase balanced system, is considered. The interconnection between transmission system and distribution system is via boundary nodes. An approximately linearized *DistFlow* model is adopted from [5] in the second-stage distribution system problem. Another alternative model convexified the AC power flow in [6], which used second-order cone to relax the quadratic constraints. Developing a convex approximation or relaxation serves for the convenience of computing subgradients when generating affine cutting planes later. Both of the models have been tested in the numerical experiment to compare the accuracy. A generalized multi-cut L-shaped method is also devised in this literature to decentralize the overall solution of the two-stage uncertainty-based problem. More details about the decentralization can be found in Section IV.

The rest of the paper is fourfold. Section III provides the mathematical formulation including the linearized *DistFlow* and decomposition structure of the T-D coordinated ED; Section IV illustrates the solution paradigm of the generalized multi-cut L-shaped method. Results of numerical experiments on a *Tran6Dist7+9* system are discussed in Section V; Section VI summarizes the paper and presents some future potentials on this topic.

III. Mathematical Formulation

A. Stochastic Expected Formulation

The overall formulation of the stochastic multi-period T-D coordinated ED is shown in (1). $\mathbf{TC}$ and $\mathbf{DC}$ represent costs for the transmission part and distribution part, which are detailed in (1a) and (1b) respectively. In each cost formulation, the piecewise linear thermal generation curve is utilized and load shedding is permitted. Note that in the distribution-side cost, the optimization variables are associated with scenario index ω_d. All of the uncertainties in the model are restricted to the distribution part, which is reasonable when the fluctuation of load profiles always happens in distribution systems. For the distribution part, v_d stands for the weight of different distribution systems which pertains to the importance of that distribution system. For instance, hospitals and military regions require higher priority in terms of electricity serving, which results in higher weight v_d in this model.

$$\min_{P_{g^T,h}, s_{\ell^T,h}} \mathbf{TC} + \sum_{d}^{D} v_d \min_{P^\omega_{g^D,h}, s^\omega_{\ell^D,h}} \mathbb{E}_\omega \Big\{ \mathbf{DC} \Big\}$$

subject to

$$\mathbf{TC} = \sum_{h}^{H} \Big\{ \sum_{g^T}^{G^T} \sum_{k}^{K} (a^k_{g^T} P_{g^T,h} + b^k_{g^T}) + \sum_{\ell^T}^{L^T} PC_{\ell^T} s_{\ell^T,h} \Big\}, \tag{1a}$$

$$\mathbf{DC} = \sum_{h}^{H} \Big\{ \sum_{g^D}^{G^D} \sum_{k}^{K} (a^k_{g^D} P^\omega_{g^D,h} + b^k_{g^D}) + \sum_{\ell^D}^{L^D} PC_{\ell^D} s^\omega_{\ell^D,h} \Big\}, \tag{1b}$$

$$\sum_{g^T|C(g^T)=n^T}^{G^T} P_{g^T,h} - \sum_{i|C(i)=n^T}^{I} P_{i,h} - \sum_{f^T|S(f^T)=n^T}^{F^T} P_{f^T,h} + \sum_{f^T|M(f^T)=n^T}^{F^T} P_{f^T,h} = \sum_{\ell^T|C(\ell^T)=n^T}^{L} \{ \ P_{\ell^T,h} - s_{\ell^T,h} \ \}, \ \forall n^T, \forall h, \tag{T-1c}$$

$$P_{f^T,h} = x^{-1}_{f^T} [\delta_{n^T|S(n^T)=f^T,h} - \delta_{n^T|M(n^T)=f^T,h}], \ \forall f^T, \forall h, \tag{T-1d}$$

$$P^{\min}_{g^T,h} \le P_{g^T,h} \le P^{\max}_{g^T,h}, \ \forall g^T, \forall h \tag{T-1e}$$

$$-RD_{g^T} \Delta h \le P_{g^T,h+\Delta h} - P_{g^T,h} \le RU_{g^T} \Delta h, \ \forall g^T, \forall h, \tag{T-1f}$$

$$\sum_{g^D|C(g^D)=n^D}^{G^D} P_{g^D,h} + \sum_{r|C(r)=n^D}^{R} P_{r,h}(\omega) + \sum_{i|C(i)=n^T}^{I} \xi_{i,h} P_{i,h} - \sum_{f^D|S(f^D)=n^D}^{F^D} P_{f^D,h} + \sum_{f^D|M(f^D)=n^D}^{F^D} P_{f^D,h} = \sum_{\ell^D|C(\ell^D)=n^D}^{L} \{ \ P_{\ell^D,h}(\omega) - s_{\ell^D,h} \ \}, \ \forall n^D, \forall h, \tag{D-1g}$$

$$\mathit{Distribution\ Line\ Flow\ Constraints}, \tag{D-1h}$$

$$P^{\min}_{g^D,h} \le P_{g^D,h} \le P^{\max}_{g^D,h}, \ \forall g^D, \forall h, \tag{D-1i}$$

$$-RD_{g^D} \Delta h \le P_{g^D,h+\Delta h} - P_{g^D,h} \le RU_{g^D} \Delta h, \ \forall g^D, \forall h, \tag{D-1j}$$

The constraints are divided into two groups, *i.e.* transmission part and distribution part, which are labeled with T and D in the equation tag. Specifically, constraints (T-1c) and (D-1g) are the active power balance constraints, wherein $\xi_{i,h}$ in (D-1g) is the dual variable of (T-1c) $\forall n^T \in I$ and indicates the locational marginal price (LMP) offered by the transmission side to the distribution system; the positive direction of power injection is assumed from transmission to distribution; constraints (T-1d) and (D-1h) define the line flows, wherein (T-1d) imposes DC power flow approximation in transmission network flow and (D-1h) will be explicitly explained in later sections; constraints (T-1e) and (D-1i) formulate the generation limits for thermal units, whereas constraints (T-1f) and (D-1j) introduce their ramp-up/ramp-down limits. Note that conventional thermal units still play a important role in transmission/distribution system even when the renewable penetration grows wildly nowadays. This model is also scalable to distribution-microgrid co-optimization when the weights v_d for microgrids are more practically used.

B. Convexified Distribution AC Power Flow

For constraints (D-1h), the original AC power flow in distribution lines, which is highly nonlinear and thus nonconvex, cannot be simply approximated via DC approach like in the transmission level, due to more concerns on high R/X ratio and over-voltage issue in distribution systems. Currently, there are two mainstream strategies towards tackling the nonconvexity in the AC power flow, *i.e.*, linear approximation (LA) and second-order cone programming (SOCP) relaxation. In this study, both of them are discussed in terms of algorithmic incorporation and solution quality. The detailed formulation of these two strategies are discussed below.

In the case of LA on *DistFlow* introduced by [7], equations (2) below can replace (D-1h), which regulate the voltage. Load shedding is allowed here to ensure feasibility of all scenarios, which simplifies the algorithm. The distribution system here is limited to radial. $V_{n^D}^{\min}$ and $V_{n^D}^{\max}$ are assumed to be in Range A regulation, *i.e.*, 0.95 and 1.05 *p.u.*

$$V_{[n^D|S(f^D)=n^D,h]} - V_{[n^D|M(f^D)=n^D,h]} = \frac{r_{f^D}P_{f^D,h} + x_{f^D}Q_{f^D,h}}{V_0}, \quad \forall f^D, \forall h, \quad (2a)$$

$$V_{n^D,h}^{\min} \le V_{n^D,h} \le V_{n^D,h}^{\max}, \quad \forall n^D, \forall h. \quad (2b)$$

On the other hand, in the case of SOCP relaxation, active/reactive power losses are considered [6], which are omitted in the previous LA. In this formulation, SOCP constraints (3) provide another alternative to replace constraints (D-1g)-(D-1h) and ensure higher accuracy than the previous method. Note that $V_{n^D}^2 = W_{n^D}$, and this formulation is constructed based on numerical approximations, *i.e.*, $\sin\delta_{n^D} \approx \delta_{n^D}$, $V_{S(n^D)}V_{M(n^D)} \approx 1$, which are famous and widely applied in both industry and academia.

$$\sum_{g^D|C(g^D)=n^D}^{G^D} P_{g^D,h} + \sum_{r|C(r)=n^D}^{R} P_{r,h}(\omega) + \sum_{i|C(i)=n^T}^{I} P_{i,h} - \sum_{f^D|S(f^D)=n^D}^{F^D} P_{f^D,h} + \sum_{f^D|M(f^D)=n^D}^{F^D} P_{f^D,h} - \sum_{f^D|S(f^D)=n^D}^{F^D} P_{f^D,h}^{\text{loss}} = \sum_{\ell^D|C(\ell^D)=n^D}^{L} \{P_{\ell^D,h}(\omega) - s_{\ell^D,h}\}, \quad \forall n^D, \forall h, \quad (3a)$$

$$\sum_{g^D|C(g^D)=n^D}^{G^D} Q_{g^D,h} + \sum_{r|C(r)=n^D}^{R} Q_{r,h}(\omega) + \sum_{i|C(i)=n^T}^{I} Q_{i,h} - \sum_{f^D|S(f^D)=n^D}^{F^D} Q_{f^D,h} + \sum_{f^D|M(f^D)=n^D}^{F^D} Q_{f^D,h} - \sum_{f^D|S(f^D)=n^D}^{F^D} Q_{f^D,h}^{\text{loss}} = \sum_{\ell^D|C(\ell^D)=n^D}^{L} Q_{\ell^D,h}(\omega) + b_{n^D}W_{n^D}, \quad \forall n^D, \forall h, \quad (3b)$$

$$W_{[n^D|S(f^D)=n^D,h]} - W_{[n^D|M(f^D)=n^D,h]} = 2(r_{f^D}P_{f^D,h} + x_{f^D}Q_{f^D,h}) + r_{f^D}P_{f^D,h}^{\text{loss}} + x_{f^D}Q_{f^D,h}^{\text{loss}}, \quad \forall f^D, \forall h, \quad (3c)$$

$$x_{f^D}P_{f^D,h}^{\text{loss}} = r_{f^D}Q_{f^D,h}^{\text{loss}}, \quad \forall f^D, \forall h, \quad (3d)$$

$$\frac{P_{f^D,h}^{\text{loss}} W_{[n^D|M(f^D)=n^D,h]}}{r_{f^D}} \ge P_{f^D,h}^2 + Q_{f^D,h}^2, \quad \forall f^D, \forall h, \quad (3e)$$

$$W_{n^D}^{\min} \le W_{n^D,h} \le W_{n^D}^{\max}, \quad \forall n^D, \forall h. \quad (3f)$$

IV. Decomposition Structure and Solution Paradigm

This section discusses the decomposition of the original problem (1) and the corresponding solution strategy, *i.e.* generalized multi-cut L-shaped method.

The problem can be naturally divided into two groups, *i.e.* transmission problem and distribution problem(s), which are put into each stage respectively. Some previous works like [8] used similar structures, but they did not consider stochasticity, multiple subsystems and their corresponding weights, which are important in the real world for many bilevel applications.

Since the overall system is decentralized, two individual series of problems can be set up for transmission system and distribution systems respectively, which can be solved locally and without interference.

A. Master Problem

This first-stage problem, *i.e.*, the master problem (M), contains the transmission part of the coordinated ED. Note that the complicating variable in the original model is the active injection $P_{i,h}$ in the boundary node, which forms the basis of the decentralized structure.

$$M = \min_{P_{g^T,h}, s_{\ell^T,h}} \mathbf{TC} + \eta, \quad (4)$$

subject to

$$\text{Constraints (1a), (T-1c)-(T-1h).} \quad (4a)$$

A compact form for (4) is represented in (5).

$$M = \min_{\mathbf{x},\mathbf{y}} \mathbf{c}_1^\top \mathbf{x} + \eta, \quad (5)$$

subject to

$$\mathbf{H}_1^\top \mathbf{x} + \mathbf{K}_1^\top \mathbf{y} = \mathbf{r}_1, \quad (5a)$$

$$\mathbf{A}_1^\top \mathbf{x} \le \mathbf{b}_1, \quad (5b)$$

where η is the dummy variable for lower bounding cuts, and $\mathbf{y}$ is the complicating variable. With master problem solved, transmission operator determines the optimal power injection, as well as the nodal marginal price, which is just the dual solution of the operating constraint, and transfers to the distribution systems.

B. Subproblem

the second-stage problem, *i.e.*, the subproblem (S_d), contains problem concerning each distribution system.

$$S_d = \min_{P_{g^D,h}^{\omega}, s_{\ell^D,h}^{\omega}} \mathbb{E}_\omega \Big\{ \mathbf{DC} \Big\}, \quad (6)$$

subject to

$$\text{Constraints (1b), (D-1i)-(D-1l).} \quad (6a)$$

A compact form for (6) is represented in (7).

$$S_d = \min_{\mathbf{z}} \mathbf{c}_2^\top \mathbf{z}, \quad (7)$$

Algorithm 1 Generalized Multi-cut L-shaped Method

Initialization: Set $o \leftarrow 0$, $LB_o \leftarrow -\infty$, $UB_o \leftarrow \infty$, $\mathcal{O}_o = \emptyset$. Solve problem M without η to obtain initial $\mathbf{y}^{o*}$.

Step 1. $o \leftarrow o + 1$.

for $d \in D$ **do**:

Step 2. Setup and solve the subproblem (S_d) with $\mathbf{y}^*$ and known ω_d as inputs. Obtain the optimal dual vector $\boldsymbol{\gamma}_d^o(\omega_d)$ for each realization.

Step 3. Compute the coefficient vectors of the affine cut as ($\Pr_{\omega_d}$ is the probability of scenario ω_d)

$$\alpha_d^o = \sum_{\omega_d} \Pr_{\omega_d} \boldsymbol{\gamma}_d^o(\omega_d)^\top \mathbf{r}_2; \qquad \boldsymbol{\beta}_d^o = -\sum_{\omega_d} \Pr_{\omega_d} \mathbf{H}_2^\top \boldsymbol{\gamma}_d^o(\omega_d).$$

Thus the affine cut can be defined as $\alpha_d^o + (\boldsymbol{\beta}_d^o)^\top \mathbf{y} \le \eta$. Add the affine coefficient pair to the collection $\mathcal{O}_o = \mathcal{O}_{o-1} \cup (\alpha_d^o, \boldsymbol{\beta}_d^o)$.

end for

Step 4. Update $UB_o = \min\{UB_{o-1}, \mathbf{c}_1^\top \mathbf{x} + [\alpha_d^o + (\boldsymbol{\beta}_d^o)^\top \mathbf{y}]\}$ and $LB_o = \eta^o$. If $\left|\frac{UB_o - LB_o}{UB_o}\right| \le \varepsilon$, then terminate, declare optimality and report $(\mathbf{x}^o, \mathbf{y}^o)$.

Step 5. Setup and solve the master problem M with all cuts computed in the previous **for** loop with their weights v_d.

$$M = \min_{\mathbf{x},\mathbf{y}} \mathbf{c}_1^\top \mathbf{x} + \eta,$$

$$\text{subject to} \quad \mathbf{H}_1^\top \mathbf{x} + \mathbf{K}_1^\top \mathbf{y} = \mathbf{r}_1,$$
$$\mathbf{A}_1^\top \mathbf{x} \le \mathbf{b}_1,$$
$$\alpha_d^j + (\boldsymbol{\beta}_d^j)^\top \mathbf{y} \le \eta / v_d, \quad \forall j \in \mathcal{O}_o, \ \forall d \in D.$$

Let $(\mathbf{x}^{o+1}, \mathbf{y}^{o+1}, \eta^{o+1})$ denote the optimal primal solution.

goto Step 1.

subject to

$$\mathbf{H}_2^\top \mathbf{y}^* + \mathbf{K}_2^\top \mathbf{z} = \mathbf{r}_2 : \quad \boldsymbol{\gamma}, \tag{7a}$$
$$\mathbf{A}_2^\top \mathbf{z} \le \mathbf{b}_2, \tag{7b}$$

where $\mathbf{y}^*$ is a parameter inherited from the optimal complicating variable in the master problem. With subproblems solved, distribution operators obtain their optimal dispatches based on their own uncertainty sets, and give back their optimal boundary power injections via generating an affine cut by subgradients $\boldsymbol{\gamma}$ to the transmission operator. Ultimately, upon convergence, the optimality for all systems is guaranteed as no infeasibility is raised and global convexity remains strict.

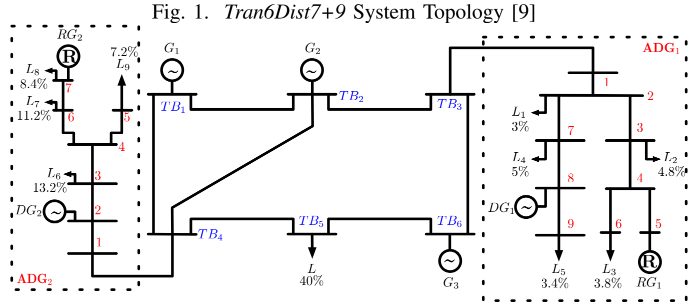
1

C. Generalized Multi-cut L-shaped Algorithm

To cope with this master-sub structure, a generalized multi-cut L-shaped algorithm is devised. The general procedure is demonstrated in Algorithm 1. Basically, with an initial point of the complicating variables, subproblem can deliver the optimal complicating variables back to the master problem via creating lower bounding affine cuts by subgradients. Each subsystem can have its own scenario set and thus return its own single-cut, which results in a multi-cut per iteration in the master system. Along with the iteration, due to convexity, those cuts form tight lower bounds to the master problem and obtain the optimality upon convergence with polynomial time if the scenario sets for all subsystems are finite.

One important issue is the cut generation under the SOCP formulation of AC power flow in the distribution systems. Upon observation on constraint (3e), however, it is clear that no complicating variable, *i.e.*, $P_{i,h}$ is encompassed, which means the SOCP constraints merely influence the second-stage distribution system. Specifically, all elements of the technology matrix $\mathbf{H}_2$ and vector $\mathbf{r}_2$ for the SOCP part are zero.

Fig. 1. *Tran6Dist7+9* System Topology [9]



The convergence proof of this algorithm is omitted due to the space limitation. The computational efficiency is based on the choice of ε and quality of scenario set.

V. Numerical Experiments

To test this strategy, case studies on a modified *Tran6Dist7+9* test system from [9] are carried out for day-ahead 24h ED problem. The system topology is depicted in Fig. 1, which is consisted of one transmission system (TS) and two radial active distribution grids (ADGs). The resistance and reactive demand/generation are not considered in [9], but they can be obtained via line R/X ratio (assume all distribution lines are with the same configuration and R/X ratio is 1:2.1) and power factor (assume to be V-I 0.95 lagging) using existed data. Ramp-up and ramp-down limits are all 40MW, and the total number of intervals K for piecewise linearization on thermal generation is 5. All of the experiments were implemented in GAMS [10] and solved by CPLEX 12.8 on a workstation of Windows PC with 8-core Intel i7-6700 CPU and 8GB of RAM.

For uncertainty, two renewable generators RG_1 in ADG$_1$ and RG_2 in ADG$_2$ are incorporated, whose active power outputs are uncertain and sampled from two different historical datasets obtained in NREL [11]. The hourly load profile is also assumed to be volatile which respects the demand behavior in NREL. In order to greatly reduce the load shed, the penalty cost for shedding is set \$1600/MW.

Firstly, 240 and 160 samples for the renewable generators RG_1 and RG_2 and 180 sample sets for all loads are generated based on the historical data. the Fast Forward/Backward approach incorporated in GAMS SCENRED toolbox is used to reduce the overall renewable generator scenarios into 10 for RG_1 and 12 for RG_2, and overall load scenarios into 10, which results in 100 hour-dependent scenarios for distribution system ADG_1 and 120 hour-dependent scenarios for distribution system ADG_2 in total. Note that the demand L in the TS is deterministic, and uncertainties only appear in the distribution parts. v_d for both distribution systems are set 1 in this study. Since it is a relatively small test system, the criterion ε is set 0, which means convergence holds when $UB_o = LB_o$.

Two comparative experiments on isolated ED and coordinated ED are carried out and reported in Table I. IED_1 mode means that TS forecasts the boundary power demand as all demands in the ADG, while IED_2 mode forecasts it as the maximum forecast demand minus the maximum generation

TABLE I
COMPARATIVE ANALYSIS ON ISOLATION AND COORDINATION

IED_1	**TS**	Costs:	$76913.86
		ADG$_1$	**ADG$_2$**
	Power Mismatch	14.8%	28.1%
	Generation Costs	$6043.27	$9652.77
	Received LMP	17.91	17.91
IED_1	**TS**	Costs:	$64146.11
		ADG$_1$	**ADG$_2$**
	Power Mismatch	3.6%	6.8%
	Generation Costs	$6243.71	$9643.86
	Received LMP	13.44	13.44
$TDCED$	**TS**	Costs:	$63392.43
		ADG$_1$	**ADG$_2$**
	Power Mismatch	0%	0%
	Generation Costs	$6236.84	$9662.21
	Received LMP	13.90	13.90

Fig. 2. Results for Linear Approximation Case

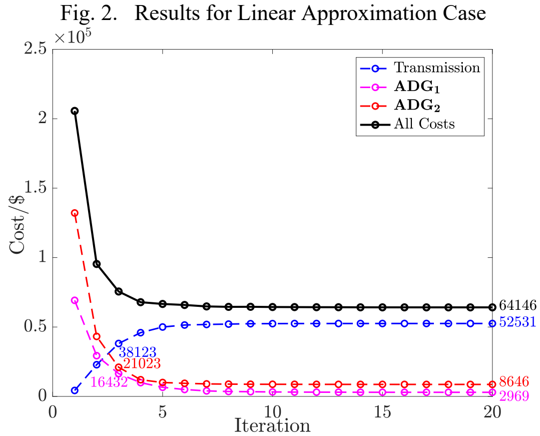
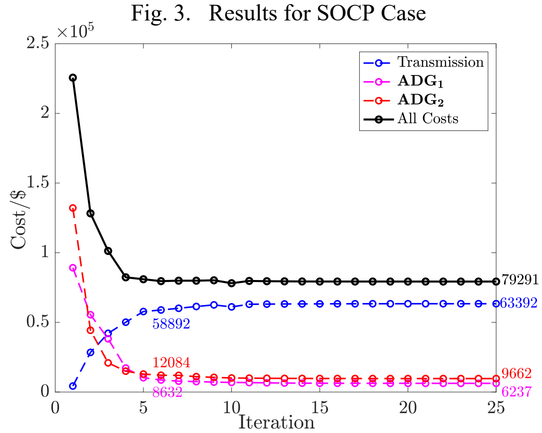


Fig. 3. Results for SOCP Case



outputs in the ADGs, and $TDCED$ stands for the coordinated ED setting, then TS performs a transmission ED [2]. The tests are based on SOCP formulation. It can be observed from the table that in both isolated modes there exists notable power mismatch, even if IED_2 mode, which is widely used in the industry, can fairly approximate the coordination as in the $TDCED$ mode. A small power mismatch, however, can still raise severe predicament for system operators. This defends the necessity of considering T-D coordination.

Fig. 2 and 3 show the results of optimal objective values in the case of using LA and SOCP for the AC power flow, respectively. Total load sheds in both cases upon convergence are within 2.7%. The costs of two ADGs start at extremely high position in that TS would like to draw huge amount of energy from the distribution sides to reduce its own power outputs in the first iteration, when ADGs have to feed TS even with high load sheds. But finally the injection becomes positive and high.

As described in section III, SOCP provides more accurate approximation on the distribution line flow, which also complicates the system. Thus, the costs in the SOCP case are higher as well as the iteration number, compared with the LA case. The average P_i and Q_i from transmission to distribution upon convergence are 74.33MW and 44.89MVar. P_{loss} and Q_{loss} in all lines are within 5%, except for some scenarios when they reach 15%, which might results from extreme cases.

Additionally, the numbers of the total constraints and variables are 166,542 and 94,420, respectively, in the LA case, while the numbers increase to 335,213 and 123,349 in the SOCP case. The average CPU time was 3.424 seconds. A centralized deterministic equivalent stochastic programming problem was also set up and solved for 5.493 seconds. Note that these numbers steam from a relatively small case, whose gap would rise dramatically in larger cases and larger scenario set. This behavior emphasizes the necessity to harness decomposition technique for computational enhancement.

VI. CONCLUSION AND FUTURE WORKS

A decentralized paradigm for coordinated economic dispatch between transmission and distribution networks is proposed in this paper. Numerical experiments show that the proposed strategy is potent and effective for this problem. Larger cases should be tested in future works, where the feasibility cut of the L-shaped method should be incorporated for seeking more efficient performance.

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